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COMPUTER TECHNOLOGIES OF DESIGN OF THE PHASE PORTRAITS OF DYNAMIC SYSTEMS

G.A. Rustamov[1], U.Kh.Agayev[2], A.T. Mammadova[2]

[1]Azerbaijan Technical University, Baku, Azerbaijan
[2]Sumgait State University, Sumgait, Azerbaijan
e-mail: gazanfar.rustamov@gmail.com

Abstract. Unfortunately MatLab and Simulink do not contain universal procedures for the design of phase portraits of dynamical systems, so one have to develop them himself. The article considers the Matlab/Simulink technology of design of phase portraits of dynamic systems and objects. The main task is to make phase portraits more prominent and informative by increaing the number of trajectories. Since manual entering of the initial conditions in Simulinik is limited and tedious automation of this process by M-files or software using MATLAB is an important practical significance. The examples are given demonstrating the efficiency of the proposed method.

Keywords: dynamic object, phase portrait, Matlab/Simulink, M-file, mathematical dancer

1. The classic methods of design of the phase portraits

The phase portrait is a set of phase trajectories. Since in the nonlinear sytems the number of equilibrium points is not uniquie as in linear systems the phse trajectories must cover all bait domains. Additionaly to investigate adequately the equilibrim points and the dynamic characteristics of the system their density should be sufficient.

Two methods are widely used to design phase portraits [1, 2]:

- Analytical method;
- Graphanalytic izoklin method.

These methods usually are applicable for the two dimensional systems.

Suppose that free motion of the two dimensional object (n=2) is geven by the equation

$$\ddot{y} + f(y, \dot{y}) = 0.$$

Making substitutions $x_1 = y$, $x_2 = \dot{y}$ one can reduce this equation to the equivalent system of equations written in state coordinates

$$\frac{dx_1}{dt} = f_1(x_1, x_2), \qquad \frac{dx_2}{dt} = f_2(x_1, x_2). \tag{1}$$

The initial conditions $x_1(0) = x_{10}$, $x_2(0) = x_{20}$ are not zero and so generate free motion.

Here x_1, x_2 are phase or state coordinates; f_1, f_2 are nonlinear functios i general case.

1.1. Analytic method. If an analytic solution for the equation (1) exitst then the found solutions $x_1(t)$ and $x_2(t)$ are the equation of the phase trajectory written in parametric form.

In some cases in (1) instead of initial equations the first order equation is used obtained by dividing the first equation of (1) by the second one:

$$\frac{dx_2}{dx_1} = \frac{f_1(x_1, x_2)}{f_2(x_1, x_2)} \quad . \tag{2}$$

This is a differential equation of the phase trajectories. If the analytic solution exists, then eqaution $x_2 = \varphi(x_1, x_{10}, x_{20})$ of the phase trajectories may be obtained.

1.2. Graphical -analytic method. In order to design the phase trajectories isocline, delta-method, Pell and other graphical methods were widely used earlier. However, as a result of use of computers and numerical integration methods, these methods are losing their relevance.

Isocline method. Let's rewirite (2) as follows

$$\frac{dx_2}{dx_1} = \frac{f_1(x_1, x_2)}{f_2(x_1, x_2)} = k(x_1, x_2) \, .$$

As we see $k(x_1, x_2)$ is a angular coefficient of the tangent to the phase trajectory at the point (x_1, x_2). Since the direction of the phase trajectory at the neibourhood of the point (x_1, x_2) is close to the direction of this tangent constructing dence enough domain of tangents the phase trajectories may be desined.

To construct the domain of tangents the its eqaition may be used

$$\frac{f_1(x_1, x_2)}{f_2(x_1, x_2)} = k \, . \tag{3}$$

Giving different values to the variables x_1 and x_2 from the proper domain one can calculate k and then drow from (x_1, x_2) a little interval of the tangent with angle $\alpha = arctg\, k$. But this way is not rational and makes difficult to follow the properties of the domain of tangents.

That is why usually giving constant values to k from the interval $k \in [-k_{\min}, k_{\max}]$ they construct the curve $x_2 = \varphi_1(x_1, k)$, $k = const$. This equation is obtained from implicit equation (3) by its solving with respect to the variable the x_2 . Therefore isocline is geometrical place of the tangents with the same inclined. Here isi stands for "constant", and cline for "tangent". The eqation of isocline is linear.

To identify the direction on each isocline curve the little parts are drown of the tangents with the same $k = const$ and so parallel. Then beginning from each isocline a parallel to the corresponding tangent is drown until crossing the next isocline and so on. So we get part wise approximation of the phase trajectory and the accuasy depends on the dencity of the isoclines.

To understant the theoretical considerations here we give an example. Consider the phase trajectory design for the twise integrable object $\dot{x}_1 = x_2$, $\dot{x}_2 = \mathrm{u}$ In this case the eqation of the isocline will be as follows

$$x_2 = \frac{\mathrm{u}}{\mathrm{k}}, \quad \mathrm{k} \neq 0$$

Take $\mathrm{u} = +1$.

In Fig 1 the corresponding isoclines (dash line) and tangent domain is given when angular coefficient is $\mathrm{k} = \{-2, -1, +1, +2\}$.

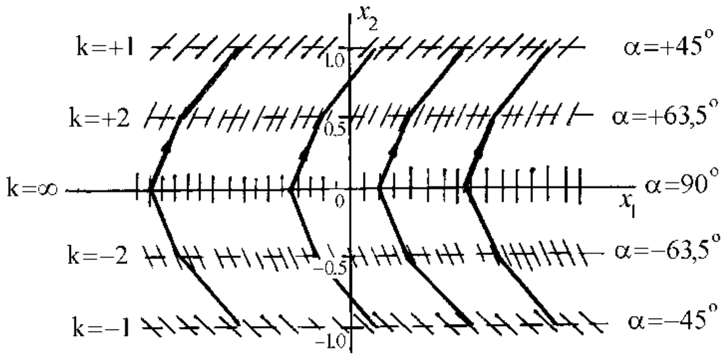


Fig. 1

As is seen from this when $\mathrm{u} = +1$ the phase trajectories are parabolas.

2. Manual input of the initial conditions in Simulink

Consider the design of the phase portraits of the mathematical dancer as a friction free object. The eqation of the object is [3]:

$d^2\theta / dt^2 = -k_1 \sin\theta, \quad k_1 = g / l.$

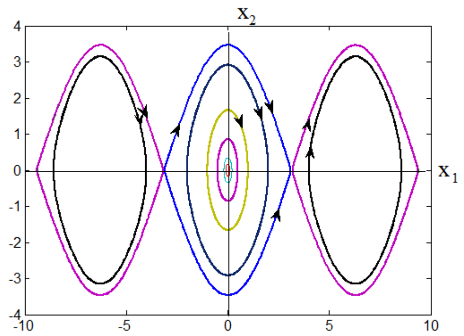
where *m* is a weigh of the load; θ is an inclination angle from the stability state; *l* – lenth of the bar.

Denoting $y = x_1 = \theta, x_2 = \dot{y} = \dot{\theta}$ we obtain the state eqation

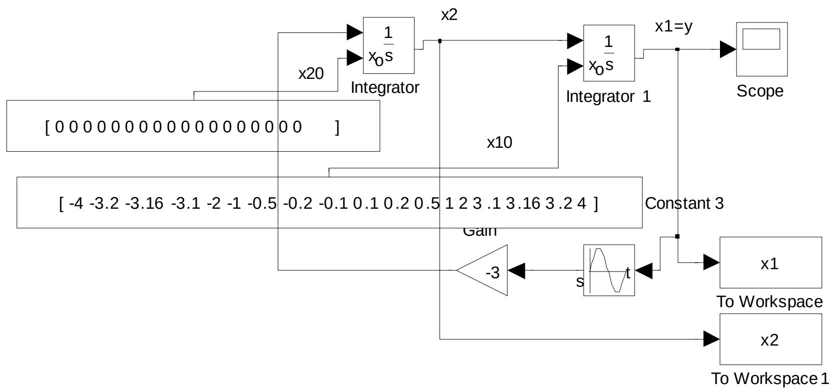
$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -k_1 \sin x,$$
$$y = x_1.$$

Since the friction is absent the stasionaries of this object consist “center” type equilibruim points.

In Fig. 2 the Simulink scheme of the solution of the equation is given for the 18 values of the initial conditions x_{10} and x_{20} at *a* and *b*. The phase portrait is obtained by the procedure *plot*(x1, x2) of the MATLAB procedure icon. As we see in spite of use of 18 initial points the phase portraint is obtained very sparse. Adding initial points covering neibourhood of the equilibruim points one can increase the density of the phase trajectories. But this is very complicated task.



a)



b)

Fig. 2

The modes of integrater (to open the parameters icon by twise clicking): External reset-**none**; Initial condition source-**external** (Fig. 1,2) or **internal**. In the last mode the initial conditions may be input manually to the parameters icon [-4 1 3...]. If to use M-files then in the first integrater one should write x10, and in the second x20. Note that in the manual input case the number of the initial conditions may be equal or ina any integrater a single number.

The mode of Workspace block:Variable name-**x1**, in the second block **x2**;Save format-**Array**.

Since the linear systems have the uniquie equilibruim state in the origin the manual editing of the initial conditions in this case may be expedient.

Let's consider an example. Bir misala baxaq.

Suppose we have the second order aperiodic object.

$$W = \frac{1}{s^2 + 2.5s + 1}.$$

The roots of the characteristic equation indeed are: p_1=-2, p_2=-0.5.

In Fig.3 a phase portraint is given designed by the 16 initial states. The solving scheme in Simulink corresponds to Fig.2.

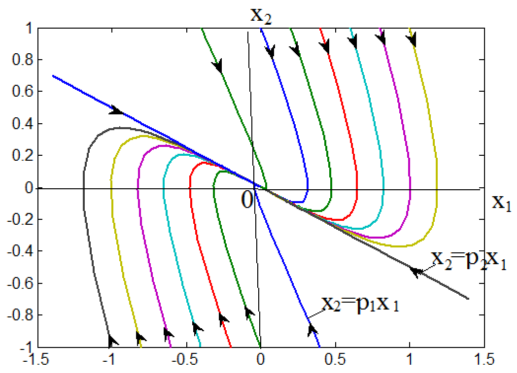


Fig. 3

Since the phase trjectories do not change inside of the asymptots $x_2 = -0.5x_1, x_2 = -2.5x_1$ it is not possible to investigate this area. This defect may be eleminated by increasing the number of the initial conditions by manually.

3. Automation the design of phase portraits in Simulinke using M-Files

By automation of the gerenration of the conditions (for example, as a random combinations in the area of separated attraction domains or changing of the initial conditions with small steps in the M-files) the considered manual technology can improved significantly.

Let's consider this technology on the example of design of phase portrait of the mathematical dancer with friction. The state equation of the object is

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = -\sin x_1 - 0.5x_2,$$

$$y = x_1.$$

Since here the term $0.5x_2$ generates a friction, in differ from the previous example the type of the equilibruim state of this object is "stable focus" (stable vibration process).

The design of the phase portrait using M-files contains the following steps:

1. The Simulink scheme of the solution of the equation is constructed following 1 and 2 (Fig.4).

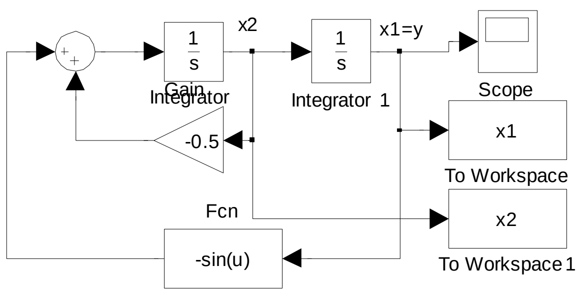
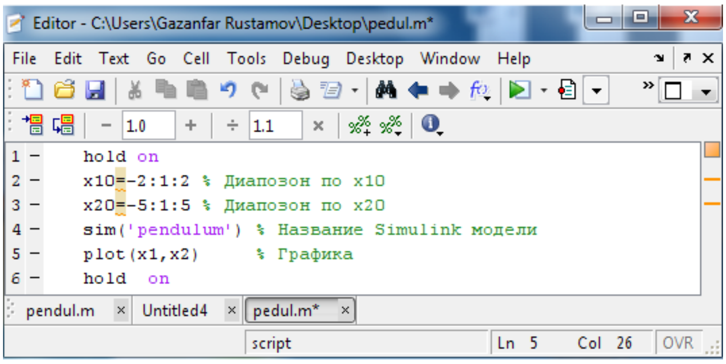


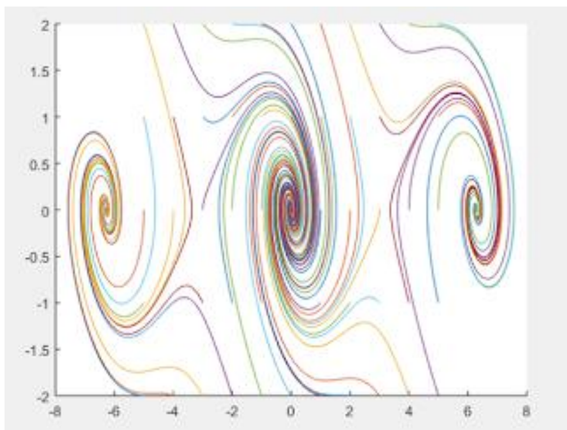
Fig. 4

The Matlab variables x10 and x20 are written that will be automatically changed taking the previous rejims in integrater In the To Workspace blocks in the **Array** mode instead of **simout** x1 and x2 is written. Saving the title is given (in our case **pendulum**).

2. To automate the design of the phase portrait the following M-code can be used.



3. After activation of the programm the phase portrait will be displayed (Fig. 5).



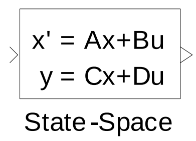


Fig.5 Fig. 6

The phase portrait obtained following to the steps and intervvals of the initial conditions input by the M-files consists of the 55 phase trajectories. The number of the trajectories may be increased if it is necessary.

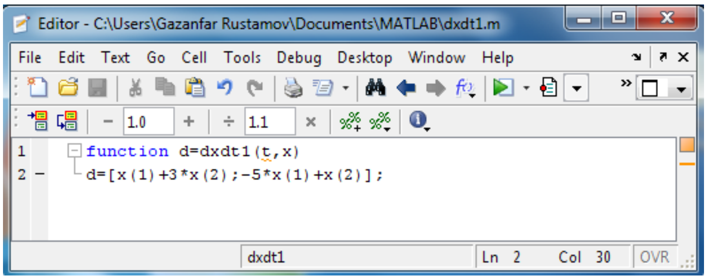
Note that for the automation of the design of the phase portrait the state model of the object given in thy matrix form also may be used (Fig. 6).

4. Programming in MATLAB

Now let’s consider design of the phase portrait by immediate programming in MATLAB. The object consists of connected system of linear equations:

$$dx/dt = x + 3y,$$
$$dy/dt = -5x + 2y.$$

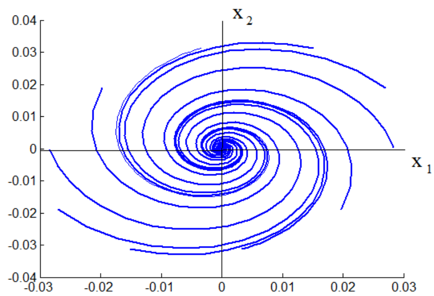
First the right hand side of the differential equation in M-file is written



The phase portrait consisting of 11 trajectories and the program written in the Matlab procedure icon is given in Fig. 7 *a* and *b*.

```
>> figure (1)
>> hold on
>> for theta =[0:10]*pi/5
x0=1e-5*[cos(theta);sin(theta)];
[t,x]=ode45(@dxdt1,[0 8],x0);
plot(x(:,1),x(:,2))
end
>>
```

a)



b)

Fig. 7

Since in Fig.7,*b* the direction of the phase velocity shown by arrows is not given it is impossible to define the type of the equailibruim state. So it is necessary to make additional research. In the case of linear object the type of the equilibruim state may be defined by the roots of the characteristic equation $\det(sI - A) = 0$ (the eigenvalues of the Matrix *A*). Using the function **eig(A)** we find in Matlab the eigenvalues as

```
>> A=[1 3;-5 2];
>> eig(A)

        1.5000 + 3.8406i
ans =
        1.5000 - 3.8406i
```

Since the real parts o0f the roots are greater that zero the equilibruim state is unstable, and is type “unstable focus” i.e. the considered process is unstable vibration process.

5. Conclusuion

At present, standard and universal tools of computer technologies for the design of the phase portraits in Matlab and Simulink is not developed enough. So the authors develop variuous algorithms for this purpose. Proposed in the work method may be applied for the design of the phase portrait of any dynamic object. The main defect is significant human participation in the design of the qualitative phase portrait of the uncertain and chaotic systems.

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